AFM Notes, Unit 1 - Probability
1-3 Factorial Notation

Name _____
Date 2-3-2020 Period 2

Ex. 1) I have eight books I want to arrange on a shelf.

a) How many different ways can I arrange the eight books?

1) Using the permutations operation on the calculator $8P_8 = 40{,}320$
2) Using the Fundamental Principle of Counting

$$8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 40{,}320$$

A third way to express this answer is by using factorial notation:

$$8! \Rightarrow 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$$

b) What if I only want to arrange 3 of my 8 books on a shelf? How many ways can I do this?

Again, we've already discussed two ways to calculate the answer to this problem.

1) Using the permutations operation on the calculator $8P_3 = 336$
2) Using the Fundamental Principle of Counting

$$8 \cdot 7 \cdot 6 = 336$$

We can also express this answer by using factorial notation

$$\frac{8!}{5!} = \frac{8 \cdot 7 \cdot 6 \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}} \qquad {}_8P_3 = \frac{8!}{(8-3)!} = \frac{8!}{5!} = 336$$

This last expression is actually the formula for a permutation. If we want to calculate the number of permutations of n objects taken r at a time, we would write:

$$P_{n\,r} = \frac{n!}{(n-r)!}$$

Ex. 2) Calculate the expression 120!/116!

$$\frac{120 \cdot 119 \cdot 118 \cdot 117 \cdot \cancel{116} \cdot \cancel{115} \ldots}{\cancel{116} \cdot \cancel{115} \ldots} = 197{,}149{,}680$$

Ex. 3) Calculate the expression 76!/73!

$$\frac{76 \cdot 75 \cdot 74 \cdot \cancel{73} \cdot \cancel{72} \cdots}{\cancel{73} \cdot \cancel{72} \cdots} = 421{,}800$$

Ex. 4) Calculate the expression n!/(n-3)!

$$\frac{n!}{(n-3)!} = \frac{n \cdot (n-1) \cdot (n-2) \cdot \cancel{(n-3)} \cdot \cancel{(n-4)} \cdot \cancel{(n-5)} \cdots}{\cancel{(n-3)}\ \cancel{(n-4)}\ \cancel{(n-5)} \cdots}$$

$$= n(n-1)(n-2) \Rightarrow n(n^2-2n-1n+2) \quad \text{FOIL}$$

$$n(n^2-3n+2)$$

$$\boxed{n^3-3n^2+2n}$$

	n	-1
n	n^2	$-1n$
-2	$-2n$	2

Ex. 5) Calculate the expression ${}_nP_{n-3}$

$$P_{n\,r} = \frac{n!}{(n-r)!} \qquad P_{n\,n-3} = \frac{n!}{(n-(n-3))!}$$

$$= \frac{n!}{3!} = \boxed{\frac{n!}{6}}$$

$$C_{n\,r} = \frac{n!}{(n-r)!\,r!}$$

Repeated Letters:

$$\frac{n!}{n_1! \cdot n_2! \cdot n_3! \cdots}$$

$n!$ → total amount of letters

$n_1!, n_2!, n_3! \ldots$ → repeating letters

LETTER → $\frac{6!}{(2! \cdot 2!)} = 180$

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