

A compound probability is a probability involving TWO OR MORE events, for example, the probability of Event A AND Event B happening.

AND  $\Rightarrow$  multiply

### Example 1: Coin Flip

What's the probability of flipping a coin twice and having it come up heads both times?

$P(\text{Comes up heads}) = \frac{\# \text{ of events}}{\# \text{ possible outcomes}} \cdot \frac{\# \text{ of events}}{\# \text{ possible outcomes}}$   
 $P(\text{1st flip comes up heads AND 2nd flip comes up heads})$

$$\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4} = 25\%$$

### The Multiplication Rule

When calculating the probability of two events, Event A and Event B, if the events are dependent, then the probability of both events happening is  $P(A) \cdot P(B)$

### Compound Probability and Replacement

#### Example 2: Marbles

A bag contains 4 blue marbles, 6 green marbles and 3 yellow marbles. If two marbles are drawn at random from the bag, what's the probability of:

- a) First drawing a green marble, and then drawing a yellow marble? Dependent  
With replacement    Independent    Without replacement

$$P(\text{G and Y}) =$$

$$\frac{6}{13} \cdot \frac{3}{13} = \frac{18}{169} = 0.106 = 10.6\%$$

- b) Drawing two blue marbles?

With replacement

$$P(\text{B and B}) =$$

$$\frac{4}{13} \cdot \frac{4}{13} = \frac{16}{169} = 9.5\%$$

$$P(\text{G AND Y}) =$$

$$\frac{6}{13} \cdot \frac{3}{12} = \frac{18}{156} = 0.115 = 11.5\%$$

Without replacement

$$P(\text{B and B}) =$$

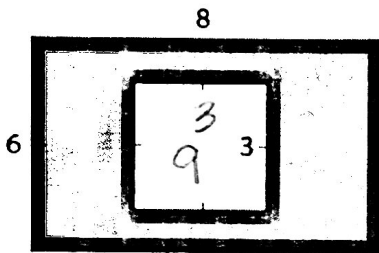
$$\frac{4}{13} \cdot \frac{3}{12} = \frac{12}{156} = 7.7\%$$

Geometric Probability - a probability that is found by calculating a ratio of lengths or area of a geometric figure.

$$P(\text{Event}) = \frac{\text{Event Area}}{\text{Total Area}}$$

Example 3: Geometric Probability of Rectangles  $A = l \cdot w$

a) What is the probability that a point chosen at random in the rectangle will also be in the square.



$$P(\text{point in square}) = \frac{9 \rightarrow \text{area of square}}{48 \rightarrow \text{area of total region}} = 18.8\%$$

(b) What is the probability that a point chosen at random in the rectangle will be in the shaded area?

$$P(\text{shaded region}) = \frac{48 - 9}{48} = \frac{39}{48} = 81.3\%$$

Example 4: Geometric Probability of Circles

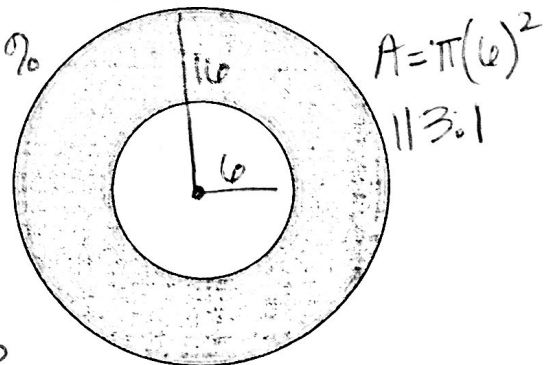
The radius of the inner circle is 6cm, and the radius of the outer circle is 16cm. Find the probability that a point selected at random in the outer circle will be in the

$$A = \pi r^2$$

$$A = \pi (16)^2 = 804.2$$

(a) inner circle

$$P(\text{inner circle}) = \frac{113.1}{804.2} = 0.141 = 14.1\%$$



(b) shaded area

$$P(\text{shaded}) = \frac{804.2 - 113.1}{804.2} = 0.859 = 85.9\%$$

## Mutually Exclusive Events

When you roll a die, an event such as rolling a 1 is called a simple event because it consists of only one event.

An event that consists of two or more simple events is called a compound event. Such as the event of rolling an odd number or a number greater than 5.

Mutually Exclusive Events is when two events cannot occur at the same time. Like the probability of drawing a 2 or an ace is found by adding their individual probabilities.

If two events, A and B, are mutually exclusive, then the probability of A or B occurs is the sum of their probabilities.

$$P(A \text{ or } B) = P(A) + P(B)$$

## Example 5: Two Mutually Exclusive Events

Keisha has a stack of 8 baseball cards, 5 basketball cards, and 6 soccer cards. If she selects a card at random from a stack, what is the probability that it is a baseball or a soccer card?

$$P(B \text{ or } S) = \frac{8}{19} + \frac{6}{19} = \frac{14}{19} = 0.736 = 73.6\%$$

## Example 6: Three Mutually Exclusive Events

There are 7 girls and 6 boys on the junior class homecoming committee. A subcommittee of 4 people is being chosen at random to decide the theme for the class float. What is the probability that the subcommittee will have a least 2 girls?

→ combinations

$$\frac{\binom{7}{2} \cdot \binom{6}{2}}{\binom{13}{4}} + \frac{\binom{7}{3} \cdot \binom{6}{1}}{\binom{13}{4}} + \frac{\binom{7}{4} \cdot \binom{6}{0}}{\binom{13}{4}} =$$

$$\frac{21 \cdot 15}{715} + \frac{35 \cdot 6}{715} + \frac{35 \cdot 1}{715} = \frac{560}{715} = 0.783$$

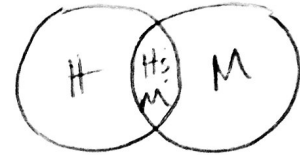
$$\frac{315}{715} + \frac{210}{715} + \frac{35}{715} = 78.3\%$$

## Inclusive Events

Since it is possible to draw a card that is both queen and a diamond, these events are not mutually exclusive, they are inclusive events.

If two events, A and B, are inclusive, then the probability that A or B occurs is the sum of their probabilities decreased by the probability of both occurring.

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$



## Example 7: Education

The enrollment at Southburg High school is 1400. Suppose 550 students take French, 700 take algebra, and 400 take both French and algebra. What is the probability that a student selected at random takes French or algebra?

$$P(\text{French}) + P(\text{Algebra}) - P(\text{French} \cap \text{Algebra})$$

$$\frac{550}{1400} + \frac{700}{1400} - \frac{400}{1400} = \frac{17}{28} = 0.607 = 60.7\%$$

## Conditional Probability

The probability of an event under the condition that some preceding event has occurred is called conditional probability. The conditional probability that event A occurs given that event B occurs can be represented by  $P(A|B)$ .

The conditional probability of event A, given event B, is defined as 
$$\frac{P(A \text{ and } B)}{P(B)}$$

## Example 8: Medicine

Refer to the application below. What is the probability that a test subject's hair grew, given that he used the experimental drug?

|                | Number of Subjects |               |
|----------------|--------------------|---------------|
|                | Using Drug         | Using Placebo |
| Hair Growth    | 1600               | 1200          |
| No Hair Growth | 800                | 400           |

total: 4000

$$P(\text{hair growth} | \text{used drug}) = \frac{1600}{4000} = \frac{2400}{4000} = 67\%$$

→ hair growth in used drug  
→ used drug