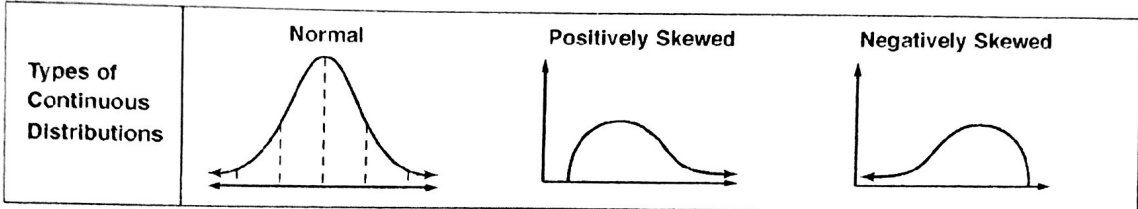


# a-4.0 Study Guide and Intervention

## The Normal Distribution

**Normal and Skewed Distributions** A continuous probability distribution is represented by a curve.

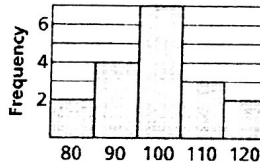


**Example** Determine whether the data below appear to be *positively skewed*, *negatively skewed*, or *normally distributed*.

{100, 120, 110, 100, 110, 80, 100, 90, 100, 120, 100, 90, 110, 100, 90, 80, 100, 90}

Make a frequency table for the data.

Value	80	90	100	110	120
Frequency	2	4	7	3	2



Then use the data to make a histogram.

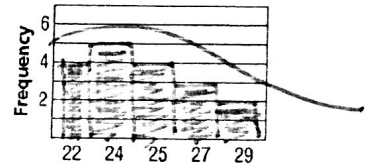
Since the histogram is roughly symmetric, the data appear to be normally distributed.

### Exercises

Determine whether the data in each table appear to be *positively skewed*, *negatively skewed*, or *normally distributed*. Make a histogram of the data.

1. {27, 24, 29, 25, 27, 22, 24, 25, 29, 24, 25, 22, 27, 24, 22, 25, 24, 22}

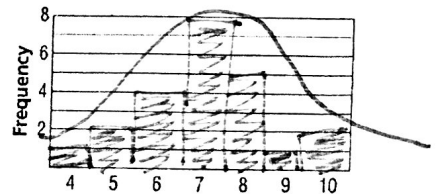
Positively skewed



2.

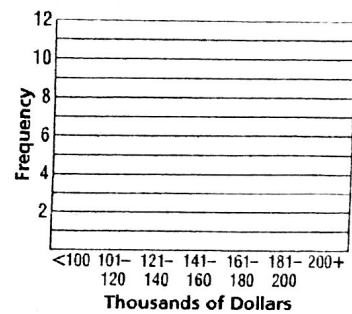
Shoe Size	4	5	6	7	8	9	10
No. of Students	1	2	4	8	5	1	2

Normal distribution



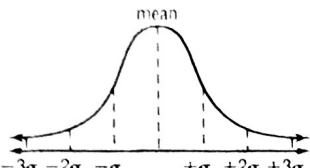
3.

Housing Price	No. of Houses Sold
less than \$100,000	0
\$100,00-\$120,000	1
\$121,00-\$140,000	3
\$141,00-\$160,000	7
\$161,00-\$180,000	8
\$181,00-\$200,000	6
over \$200,000	12



2-4.0

**Study Guide and Intervention** (continued)**The Normal Distribution****Use Normal Distributions**

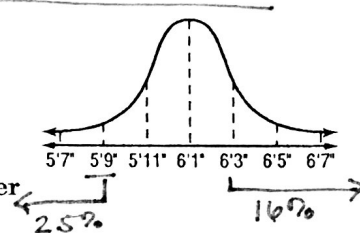
<p><b>Normal Distribution</b></p> 	<p>Normal distributions have these properties.</p> <p>The graph is maximized at the mean.</p> <p>The mean, median, and mode are about equal.</p> <p>About 68% of the values are within one standard deviation of the mean.</p> <p>About 95% of the values are within two standard deviations of the mean.</p> <p>About 99% of the values are within three standard deviations of the mean.</p>
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**Example:** The heights of players in a basketball league are normally distributed with a mean of 6 feet 1 inch and a standard deviation of 2 inches.

- a. What is the probability that a player selected at random will be shorter than 5 feet 9 inches?

Draw a normal curve. Label the mean and the mean plus or minus multiples of the standard deviation.

The value of 5 feet 9 inches is 2 standard deviations below the mean, so approximately 2.5% of the players will be shorter than 5 feet 9 inches.



- b. If there are 240 players in the league, about how many players are taller than 6 feet 3 inches?

The value of 6 feet 3 inches is one standard deviation above the mean. Approximately 16% of the players will be taller than this height.

$$240 \times 0.16 \approx 38$$

About 38 of the players are taller than 6 feet 3 inches.

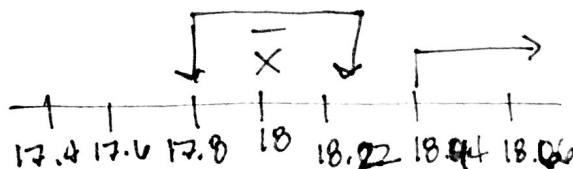
**Exercises**

**EGG PRODUCTION** The number of eggs laid per year by a particular breed of chicken is normally distributed with a mean of 225 and a standard deviation of 10 eggs.

- About what percent of the chickens will lay between 215 and 235 eggs per year? *68%*
- In a flock of 400 chickens, about how many would you expect to lay more than 245 eggs per year?  $400 \times 0.025 = 10 \text{ chickens}$

**MANUFACTURING** The diameter of bolts produced by a manufacturing plant is normally distributed with a mean of 18 mm and a standard deviation of 0.2 mm.

- What percent of bolts coming off of the assembly line have a diameter greater than 18.4 mm? *2.65% or 2.5%*
- What percent have a diameter between 17.8 and 18.2 mm? *68%*



Normal Distributions and Percentile Ranks

The Empirical Rule - States nearly all values lie within 3 standard deviations from the mean in Normal distribution.

In any 68-95-99 data set that is normally distributed:

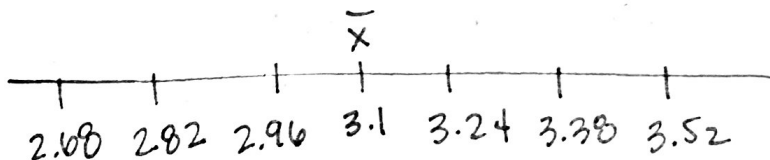
Approx. 68% of the values will be within 1 standard deviation of the mean

Approx. 95% of the values will be within 2 standard deviations of the mean

Approx. 99.7% of the values will be within 3 standard deviations of the mean

**Ex. 1:** A group of students weighs 500 US pennies. They find that the pennies have normally distributed weights with a mean of 3.1g and a standard deviation of 0.14g.

- a) Sketch the normal curve for this distribution below. Label the mean and three standard deviations above and below the mean.



- b.) What percent of the pennies have a weight that lies between:
- 2.96g and 3.24g (i.e., within one standard deviation of the mean)? 68%
- 2.82g and 3.38g (i.e., within two standard deviations of the mean)? 95%
- 2.68g and 3.52g (i.e., within three standard deviations of the mean)? 99.7%

c.) How many pennies have a weight that lies within

2.96g and 3.24g (i.e., within one standard deviation of the mean)? 340

$$500 \times 0.68$$

2.82g and 3.38g (i.e., within two standard deviations of the mean)? 475

$$500 \times 0.95$$

2.68g and 3.52g (i.e., within three standard deviations of the mean)? 499

$$500 \times 0.997$$

What if I wanted to know the percentage of pennies that had a weight between 3g and 3.2g? These values do NOT fall on normal distribution chart!

Calculator Function: normalcdf()

The TI83/TI84 calculators have a function called normalcdf() which will tell you:

the percentage of values that lie within a given  
and all you have to give it is: interval interval  
mean (x)  
standard dev. (s<sub>x</sub>)

(Note that normalcdf assumes that your data set is normally distributed.)

The format of the normalcdf() function is:

normalcdf( lower bound , upper bound , mean , stan. dev )  
interval interval

So if we wanted to know the percentage of pennies from our data set that had a weight between 3g and 3.2g, we would enter the following into our calculator:

normalcdf ( 3 , 3.2 , 3.1 , 0.14 )

2<sup>nd</sup> vars (distrib) → #2

around 53%

of our pennies  
would weigh  
between 3g and  
3.2 grams

## Percentile Ranks

A percentile is a measure that tells us what percent of the total frequency scored below that measure. A percentile rank is the percentage of scores that fall below a given score.

## About Percentile Ranks:

- Percentile rank is a number between 0 and 100 indicating the percent of cases falling at or below that score.
- Percentile ranks are usually written to the nearest whole percent: 74.5% → 75% → 75th percentile
- Scores are arranged in rank order from least to greatest.
- There is no 0 percentile rank - the lowest score is at the 1 (first) percentile.
- There is no 100 percentile rank - the highest score is at the 99th percentile.

## Consider:

1. Karl takes the big Earth Science test and his teacher tells him that he scored at the 92<sup>nd</sup> percentile. What does it mean that he scored in the 92<sup>nd</sup> percentile?

He scored better than 92% of the students who took the test.

2. Sue takes the Chapter 4 math test. If Sue's score is the same as "the mean" score for the math test, she scored at the 50<sup>th</sup> percentile. What does this mean?

She scored better than half the students who took the test.

Example 1: If Jason graduated 25<sup>th</sup> out of a class of 150 students, then 125 students were ranked below Jason. Jason's percentile rank would be:

$$\frac{125}{150} = 0.8\bar{3} \Rightarrow 83^{\text{rd}} \text{ percentile}$$

Example 2: The math test scores were: 50, 65, 70, 72, 72, 78, 80, 82, 84, 84, 85, 86, 88, 88, 90, 94, 96, 98, 98, 99. Find the percentile rank for a score of 84 on this test.

\* List data pts in order

$$\frac{CF + 0.5F}{n} \cdot 100 \quad \rightarrow \text{Frequency of given data pt}$$

$\downarrow$   
 # data pts below the data pt given

$$\frac{8 + 0.5(2)}{20} \cdot 100 = 45^{\text{th}} \text{ percentile}$$

Example 3: The math test scores were: 50, 65, 70, 72, 72, 78, 80, 82, 84, 84, 85, 86, 88, 88, 90, 94, 96, 98, 98, 99. Find the percentile rank for a score of 86 on this test.

$$\frac{11 + 0.5(1)}{20} \cdot 100 = 57.5 \rightarrow 58^{\text{th}} \text{ percentile}$$

Example 4: Find the values at the 20th and 80th percentiles for each set of values.

a. 188 168 174 198 186 170 180 182 186 176

$$\frac{168, 170, 174}{(0.20)(10) = 2} \Rightarrow 174$$

$$(0.80)(10) = 8 \Rightarrow 188$$

\* Values in order from least to greatest

b. 376 324 346 348 350 352 356 368 345 360

324, 345, 346, 348, 350, 352, 356, 360, 368, 376

$$(0.2)(10) = 2 \Rightarrow 346$$

$$(0.8)(10) = 8 \Rightarrow 368$$