AFM Notes
3-1 Exponential Functions

Name: ____________
Date: ____________ Period: ______

In an exponential function like $y = 2^x$, the base is a constant, and the exponent is a variable. Let's examine the graph of $y = 2^x$.

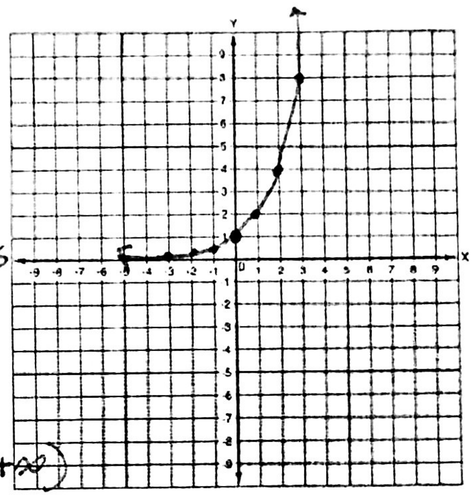
Example 1: Graph an Exponential Function

Sketch the graph of $y = 2^x$. Then state the function's domain and range.
Make a table of values. Connect the points to sketch a smooth curve.

X	Y
-3	0.125
-2	0.25
-1	0.5
0	1
1	2
2	4
3	8

* Function
* Increasing function
* y intercept (0,1)
* No x intercept
* approaches the x axis but does not touch
* asymptote y=0

Domain: $(-\infty, \infty)$ Range: $(0, +\infty)$

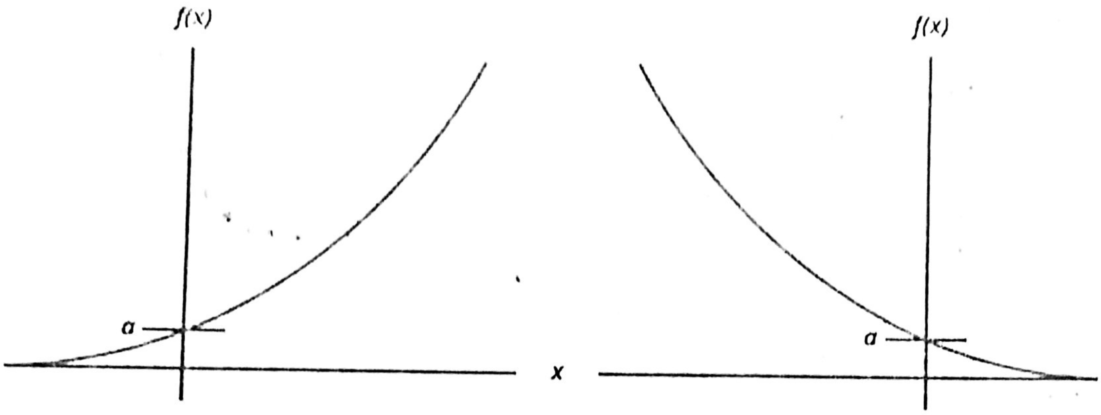


In general, an equation of the form $y = ab^x$, where $a \neq 0$, $b > 0$, and $b \neq 1$, is called an exponential function with the base b.

Exponential functions have the following characteristics:

1. The function is continuous and one to one.
2. The domain is the set of all real numbers.
3. The x-axis is an asymptote of the graph. (y=0)
4. The range is the set of all positive numbers if $a > 0$ and all negative numbers if $a < 0$.
5. The graph contains the point $(0, a)$. That is, the y-intercept is *a*.
6. The graphs of $y = ab^x$ and $y = a(1/b)^x$ are reflections across the *y-axis*.

There are two types of exponential functions: exponential growth and exponential decay. The base of an exponential growth function is a number greater than 1. The base of an exponential decay function is a number between 0 and 1.



(a) Exponential growth
$f(x) = ab^x$ with $b > 1$

(b) Exponential decay
$f(x) = ab^x$ with $0 < b < 1$

$\rightarrow y = 2(3)^x$

- If $a > 0$ and $b > 1$, the function represents exponential growth.
- If $a > 0$ and $0 < b < 1$, the function represents exponential decay.

$\rightarrow y = 100(\tfrac{1}{2})^x$

Example 2: Identify Exponential Growth and Decay

Determine whether each function represents exponential *growth* or *decay*.

a. $y = \left(\frac{1}{5}\right)^x$ $a=1$ $(0,1)$

decay

$b = \frac{1}{5}$ $\frac{1}{5} < 1$

b. $y = 3(4)^x$ $a=3$

growth $(0,3)$

$b=4$ $4>1$

c. $y = 7(1.2)^x$ $a=7$

growth $(0,7)$

$b=1.2$

$1.2>1$

Exponential functions are frequently used to model the growth or decay of a population. You can use the y intercept and one other point on the graph to write the question of an exponential function.

Example 3: Write an Exponential Function

Farming: In 1983, there were 102,000 farms in Minnesota, but by 1998, this number had dropped to 80,000.

a. Write an exponential function of the form $y = ab^x$ that could be used to model the farm population y of Minnesota. Write the function in terms of x, the number of years since 1983.

1983 → 0 $(0, 102000)$ → y intercept (inital value)

1983 → 1998 = 15 $(15, 80000)$ ↳ a

x y

$y = ab^x$

$80000 = 102000\, b^{15}$

$\frac{80000}{102000} = \frac{102000\, b^{15}}{102000}$

$\sqrt[15]{0.7843} = \sqrt[15]{b^{15}}$ $b \approx 0.98$

$\boxed{y = 102000(0.98)^x}$

$1 - 0.98 = 0.02$

↓ 2%

decaying at a rate of 2% each year

b. Suppose the number of farms in Minnesota continues to decline at the same rate. Estimate the number of farms in 2010.

1983 → 2010 = 27 yrs

↳ $x = 27$

$y = 102000(0.98)^{27}$

$y = 59,115.88$ farms

Exponential Equations and Inequalities

Since the domain of an exponential function includes irrational numbers such as $\sqrt{2}$, all the properties of rational exponents apply to irrational exponents.

Example 4: Simplify Expressions with Irrational Exponents

Simplify each expression.

a. $2^{\sqrt{5}} \bullet 2^{\sqrt{3}} = 2^{\sqrt{5}+\sqrt{3}}$

multiply like bases, add exponents

$2^{\sqrt{5}} \cdot 5^{\sqrt{3}} = 2^{\sqrt{5}} \cdot 5^{\sqrt{3}}$

$2^{\sqrt{3}} \cdot 2^{\sqrt{3}} = 2^{2\sqrt{3}}$

b. $\left(7^{\sqrt{2}}\right)^{\sqrt{3}} = 7^{\sqrt{2}\cdot\sqrt{3}} = 7^{\sqrt{6}}$

power to power, multiply exponents

The following property is useful for solving exponential equations. Exponential equations are equations in which the variable occur as the exponent.

$3^x = 15 \qquad 2^{x+1} = 9^{x-6}$

Property of Equality for Exponential Functions

Symbols: If b is a positive number other than 1, then $b^x = b^y$ if and only if x = y.

Example: If $2^x = 2^8$, then x = 8.

If the base is the same, drop base and set exponents equal to each other.

Example 5: Solve Exponential Equations

a. $3^{2n+1} = 81$

$3^{2n+1} = 3^4$

$2n+1 = 4$

$2n + \cancel{1} = 4$
$-\cancel{1} \ -1$

$\frac{\cancel{2}n}{\cancel{2}} = \frac{3}{2}$

$\boxed{n = 3/2 \text{ or } 1.5}$

$81 \to 9 \cdot 9 \to 3 \cdot 3 \ 3 \cdot 3$

$3^{(1.5)2+1} = 81$

b. $4^{2x} = 8^{x-1}$

$\left(2^2\right)^{2x} = \left(2^3\right)^{x-1}$

$2^{4x} = 2^{3x-3}$

$4x = 3x - 3$
$-3x \ -3x$

$1x = -3$

$\boxed{x = -3}$

$4 \to 2 \cdot 2 \qquad 8 \to 4 \cdot 2 \to 2 \cdot 2 \cdot 2$

The following property is useful for solving inequalities involving exponential functions

Property of Inequality for Exponential Functions

Symbols: If $b>1$, then $b^x > b^y$ if and only if $x>y$, and $b^x < b^y$ if and only if $x<y$.

Examples: If $5^x < 5^4$, then $x < 4$.

Example 6: Solve Exponential Inequalities

Solve $4^{3p-1} > \frac{1}{256}$

* Raise 256 to the -1 power to change from fraction form.

$4^{3p-1} > 256^{-1}$

$4^{3p-1} > (4^4)^{-1}$

$4^{3p-1} > 4^{-4}$

$3p-\cancel{1} > -4$

$\cancel{+1} \quad +1$

$\frac{3p}{3} > \frac{-3}{3}$

$\boxed{p>-1}$