AFM Notes
3-2 Logarithms and Logarithmic Functions

Name________ Date 3/5/20 Period 2

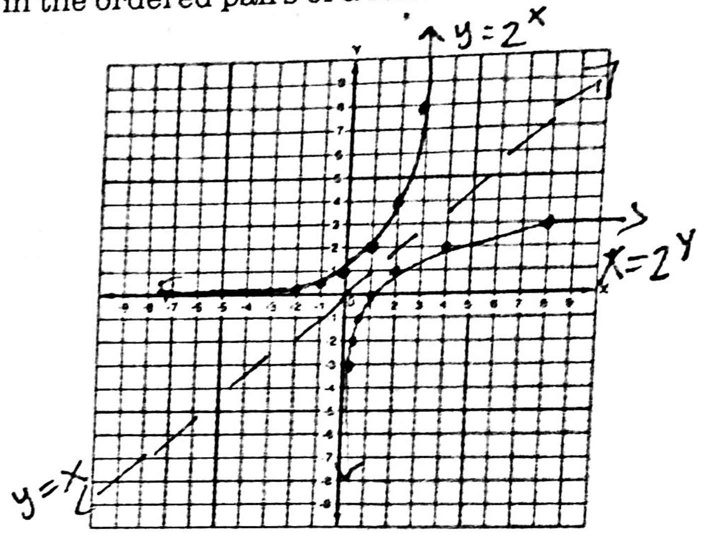
To better understand what is meant by a logarithm, let's look at he graph of $y = 2^x$ and its inverse. Since exponential functions are one to one, the inverse of $y = 2^x$ exists and is also a function. Recall that you can graph the inverse of a function by interchanging the x and y values in the ordered pairs of a function.

$y = 2^x$

x	y
-3	1/8
-2	1/4
-1	1/2
0	1
1	2
2	4
3	8

$x = 2^y$

x	y
1/8	-3
1/4	-2
1/2	-1
1	0
2	1
4	2
8	3



The inverse of $y = 2^x$ can be defined as $x = 2^y$.

Notice that the graphs of these two functions are reflections of each other over the line $y = x$. In general, the inverse of $y = b^x$ is $x = b^y$. In $x = b^y$, y is called the logarithm of x. It is usually written as $y = \log_b x$ and is read y equals log base b of x.

Logarithm with Base b

Words: Let b and x be positive numbers, $b \neq 1$. The logarithm of x with base b is denoted $\log_b x$ and is defined as the exponent y that makes the equation $b^y = x$ true.

Symbols: Suppose $b > 0$ and $b \neq 1$. For $x > 0$, there is a number y such that $\log_b x = y$ if and only if $b^y = x$.

Example 1: Logarithmic to Exponential Form

Write each equation in exponential form.

a. $\log_8 1 = 0$ $8^0 = 1$

b. $\log_2 \frac{1}{16} = -4$ $2^{-4} = \frac{1}{16}$

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$\log_{10} \Rightarrow \log$

Example 2: Exponential to Logarithmic Form

Write each equation in logarithmic form.

a. $10^3 = 1000$ $\log_{10} 1000 = 3$

$\log 1000 = 3$

b. $\cancel{9^{\frac{1}{2}} = 3}$ $9^{1/2} = 3$ $\log_9 3 = 1/2$

You can use the definition of logarithm to find the value of a logarithmic expression.

Example 3: Evaluate Logarithmic Expressions

Evaluate $\log_2 64$. what power is 2 raised to, to make 64?

$\log_2 64 = 6$

$\log_2 64 = y$ $2^y = 64$

$2^y = 2^6$ $y = 6$

Old calc

$\frac{\log 64}{\log 2} = 6$

The function $y = \log_b x$, where $b > 0$ and $b \neq 1$, is called a log function. As shown in the graph on the previous page, this function is the inverse of the exponential function $y = b^x$ and has the following characteristics:

1. The function is continuous and one to one.
2. The domain is the set of all x > 0. $(0, \infty)$
3. The y-axis is an asymptote of the graph.
4. The range is the set of all real numbers. $(-\infty, \infty)$
5. The graph contains the point (1, 0). That is, the x-intercept is 1.

Since the exponential function $f(x) = b^x$ and the logarithmic function $g(x) = \log_b x$ are inverses of each other, their composites are the identify function. That is $f(g(x)) = g(f(x)) = x$.

$f(g(x)) = x$

$f(\log_b x) = x$

$b^{\log_b x} = x$

$\log_b x = \log_b x$ ✓

$x = x$ ✓

$3^{\log_3 4} = 4$

$3^{\frac{\log 4}{\log 3}} = 4$

$g(f(x)) = x$

$g(b^x) = x$

$\log_b b^x = x$

$b^x = b^x$

$x = x$ ✓

Thus, if their bases are the same, exponential and logarithmic functions "undo" each other. You can use this Inverse property of exponents and logs to simplify expression.

Example 4: Inverse Property of Exponents and Logarithms

Evaluate each expression.

a. $\log_6 6^8 = 8$ $\log_x x^3 = 3$

$\log_4 4^7 = 7$

b. $3^{\log_3(4x-1)} = 4x-1$ $1000^{\log_{1000} 2} = 2$

$7^{\log_7 x^2} = x^2$

Solve Logarithmic Equations and Inequalities

A log equation is an equation that contains one or more logarithms. You can use the definition of logarithm to help you solve log equations.

Example 5: Solve a logarithmic Equation

Solve $\log_4 n = \frac{5}{2}$. $4^{5/2} = n$ $n = 32$

A log inequality is an inequality that involves logarithms. In the case of inequalities, the following property is helpful.

Logarithmic to Exponential Inequality

Symbols: If $b>1$, $x>0$, and $\log_b x > y$, then $x > b^y$.

If $b>1$, $x>0$, and $\log_b x < y$, then $x < b^y$. * No negative values!

Examples: $\log_2 x > 3$ * $\log_3 x < 5$

$x > 2^3$ $\boxed{x>8}$ $0 < x < 3^5$ $\boxed{0 < x < 243}$ $x>0$

Example 6: Solve a Logarithmic Inequality

Solve $\log_3 x < 2$ $0 < x < 9$

Use the following property to solve the logarithmic inequalities that have the ~~dame~~ same base on each side.

Property of Equality for Logarithmic Functions

Symbols: If b is a positive number other than 1, then $\log_b x = \log_b y$ if and only if $x = y$.

Example: If $\log_7 x = \log_7 3$, then $x = 3$.

Example 7: Solve Equations with Logarithms on Each Side

Solve $\log_5(p^2 - 2) = \log_5 p$. Check you solution.

$p^2 - 2 = p$

$p^2 - p - 2 = 0$

$\cancel{p = -1}$ $p = 2$

extraneous solution

$p^2 - p - 2 = 0$

$(p-2)(p+1) = 0$

$-2p$

$+1p$

$-1p$

$(p^2 - 2p) + (1p - 2) = 0$

$p(p-2) + 1(p-2) = 0$

$(p+1)(p-2) = 0$

Use the following property to solve logarithmic inequalities that have the same base on each side. Exclude values from your solution set that would result in taking the logarithm of a number less than or equal to zero in the original inequality.

Property of Inequality for Logarithmic Functions

Symbols: If $b > 1$, then $\log_b x > \log_b y$ if and only if $x > y$, and $\log_b x < \log_b y$ if and only if $x < y$.

Example: If $\log_2 x > \log_2 9$, then $x > 9$.

Example 8: Solve Inequalities with Logarithms on Each Side

Solve $\log_{10}(3x - 4) < \log_{10}(x + 6)$. Check your solution.

* less than!

$3x - 4 < x + 6$

$-x \quad -x$

$2x - 4 < 6$

$+4 \quad +4$

$2x < 10$

$\frac{4}{3} < x < 5$

$3x - 4 > 0$

$+4 \quad +4$

$3x > 4$

$x > \frac{4}{3}$ ✓

$x + 6 > 0$

$-6 \quad -6$

$\cancel{x > -6}$

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