AFM Notes
3-3 Properties of Logarithms

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Date____________________ Period ______

Since logarithms are exponents, the properties of logarithms can be derived from the properties of exponents.

Product Property of Logarithms

Words: The logarithm of a product is the sum of the logarithms of its factors.

Symbols: For all positive numbers m, n, and b, where $b \neq 1$.

$\log_b mn = \log_b m + \log_b n$

Example: $\log_3(4)(7) = \log_3 4 + \log_3 7$

To show that this property is true, let $b^x = m$ and $b^y = n$. Then, using the definition of logarithm, $x = \log_b m$ and $y = \log_b n$.

$$\log_2 xy = \log_2 x + \log_2 y$$

$$\underbrace{\log_3 x + \log_3 (x+2)}_{\text{Expanded}} = \underbrace{\log_3 x(x+2)}_{\text{condensed}}$$

$$\log 2 + \log 10 = \log 20$$

You can use the Product Property of Logarithms to approximate logarithmic expressions.

Example 1: Use the Product Property

Use $\log_2 3 \approx 1.5850$ to approximate the value of $\log_2 48$.

$\log_2 48 = 5.5849$

$$\log_2 48 = \log_2 (16 \cdot 3)$$

$$= \log_2 16 + \log_2 3$$

$$= \log_2 2^4 + \log_2 3$$

using properties

$$= 4 + 1.5850$$

$$= 5.585$$

Recall the quotient of powers is found by subtracting the exponents.

Quotient Property of Logarithms

Words: The logarithm of a quotient is the difference of the logarithms of the numerator and the denominator.

Symbols: For all positive numbers m, n, and b, where $b \neq 1$.

$$\log_b \frac{m}{n} = \log_b m - \log_b n$$

Example 2: Use the Quotient Property

Use $\log_3 5 \approx 1.4650$ and $\log_3 20 \approx 2.7268$ to approximate $\log_3 4$.

$$\log_3 4 = \log_3\left(\frac{20}{5}\right)$$
$$= \log_3 20 - \log_3 5$$
$$= 2.7268 - 1.4650$$

$$\log_3 4 = 1.2618$$

Example 3: Use Properties of Logarithms

The loudness L of a sound in decibels is give by $L = 10\log_{10} R$, where R is the sound's relative intensity. Suppose one person talks with a relative intensity of 10^6 or 60 decibels. Would the sound of ten people each talking at the same intensity be ten times as loud or 600 decibels?

$$\boxed{60 \cdot 10 = 600 \text{ decibels}}$$

$$60 = 10 \log_{10} 10^6$$
$$6 = \log_{10} 10^6$$
$$10^6 = 10^6$$

$$6 = \log_{10} 10^6$$

$$10^6 \cdot 10 = \cancel{10^7} = 70$$

Recall that the power of a power is found by multiplying exponents.

Power Property of Logarithms

Words: The logarithm of a power is the product of the logarithm and the exponent.

Symbols: For any number p and positive numbers m and b, where $b \neq 1$

$$\log_b m^p = p \log_b m$$

Example 4: Power Property of Logarithms

$$\log_2 x^3 = 3\log_2 x$$

Given $\log_4 6 \approx 1.2925$, approximate the value of $\log_4 36$.

$$\log_4 36 = \log_4 (6^2)$$
$$= 2\log_4 6$$
$$= 2(1.2925)$$

$$\log_4 36 = 2.585$$

Example 5: Solve Equations Using Properties of Logarithms

Solve each equation.

① take any # in front of a log and make exponent.

② condense any addition or subtraction

③ drop logs if they are the same OR re-write in exponential form

a. $3\log_5 x - \log_5 4 = \log_5 16$

$$\log_5 x^3 - \log_5 4 = \log_5 16$$
$$\log_5 \frac{x^3}{4} = \log_5 16$$

$$\frac{x^3}{4} = 16$$
$$x^3 = 64$$
$$\boxed{x = 4}$$

b. $\log_4 x - \log_4 (x-6) = 2$

$$\log_4 \frac{x}{x-6} = 2$$
$$4^2 = \frac{x}{x-6}$$
$$16 = \frac{x}{x-6}$$
$$16(x-6) = x$$
$$16x - 96 = x$$
$$-96 = -15x$$
$$\boxed{6.4 = x}$$