

3-4 Homework

Common Logarithms

Use a calculator to evaluate each expression to four decimal places.

1. $\log 101$ 2.0043 2. $\log 2.2$ 0.3424 3. $\log 0.05$ -1.3010

Use the formula $\text{pH} = -\log[H^+]$ to find the pH of each substance given its concentration of hydrogen ions.

4. milk: $[H^+] = 2.51 \times 10^{-7}$ mole per liter 6.6003
 5. acid rain: $[H^+] = 2.51 \times 10^{-6}$ mole per liter 5.6003
 6. black coffee: $[H^+] = 1.0 \times 10^{-5}$ mole per liter 5
 7. milk of magnesia: $[H^+] = 3.16 \times 10^{-11}$ mole per liter 10.5003

Solve each equation or inequality. Round to four decimal places.

8. $2^x < 25$ 9. $5^a = 120$ 10. $6^z = 45.6$
 11. $9^m \geq 100$ 12. $3.5^x = 47.9$ 13. $8.2^y = 64.5$
 14. $2^{b+1} \leq 7.31$ 15. $4^{2x} = 27$ 16. $2^a - 4 = 82.1$
 17. $9^z - 2 > 38$ 18. $5^w + 3 = 17$ 19. $30^{x^2} = 50$
 20. $5^{x^2 - 3} = 72$ 21. $4^{2x} = 9^x + 1$ 22. $2^n + 1 = 5^{2n - 1}$

Express each logarithm in terms of common logarithms. Then approximate its value to four decimal places.

23. $\log_5 12$ 24. $\log_8 32$ 25. $\log_{11} 9$
 26. $\log_2 18$ 27. $\log_9 6$ 28. $\log_7 \sqrt{8}$

29. **HORTICULTURE** Siberian irises flourish when the concentration of hydrogen ions $[H^+]$ in the soil is not less than 1.58×10^{-8} mole per liter. What is the pH of the soil in which these irises will flourish?

$\text{pH} \geq -\log[1.58 \times 10^{-8}]$ $\text{pH} \geq 7.801$

30. **ACIDITY** The pH of vinegar is 2.9 and the pH of milk is 6.6. How many times greater is the hydrogen ion concentration of vinegar than of milk?

4 times $10^{-2.9} = [H^+] = 0.00125$ $2.9 = -\log[H^+]$ $6.6 = -\log[H^+]$

31. **BIOLOGY** There are initially 1000 bacteria in a culture. The number of bacteria doubles each hour. The number of bacteria N present after t hours is $N = 1000(2)^t$. How long will it take the culture to increase to 50,000 bacteria?

$50000 = 1000(2)^t$ $t = 5.64$

32. **SOUND** An equation for loudness L in decibels is given by $L = 10 \log R$, where R is the sound's relative intensity. An air-raid siren can reach 150 decibels and jet engine noise can reach 120 decibels. How many times greater is the relative intensity of the air-raid siren than that of the jet engine noise?

$150 = 10 \log R$ $120 = 10 \log R$ 1000 times
 $15 = \log R$ $12 = \log R$
 $R = 10^{15}$ $R = 10^{12}$

$$(8) [2^x < 25] \log$$

$$x \frac{\log 2}{\log 2} < \frac{\log 25}{\log 2}$$

$$x < \frac{\log 25}{\log 2}$$

$$(9) [5^a = 120] \log$$

$$a \frac{\log 5}{\log 5} = \frac{\log 120}{\log 5}$$

$$a = \frac{\log 120}{\log 5}$$

$$(10) [6^z = 45.6] \log$$

$$z \frac{\log 6}{\log 6} = \frac{\log 45.6}{\log 6}$$

$$z = \frac{\log 45.6}{\log 6}$$

$$(11) [9^m \geq 100] \log$$

$$m \frac{\log 9}{\log 9} \geq \frac{\log 100}{\log 9}$$

$$m \geq \frac{\log 100}{\log 9}$$

$$(12) [3.5^x = 47.9] \log$$

$$x \frac{\log 3.5}{\log 3.5} = \frac{\log 47.9}{\log 3.5}$$

$$x = \frac{\log 47.9}{\log 3.5}$$

$$(13) [8.2^y = 64.5] \log$$

$$y \frac{\log 8.2}{\log 8.2} = \frac{\log 64.5}{\log 8.2}$$

$$y = \frac{\log 64.5}{\log 8.2}$$

$$(14) [2^{b+1} \leq 7.31] \log$$

$$(b+1) \log 2 \leq \log 7.31$$

$$\frac{b \log 2 + \log 2}{-\log 2} \leq \frac{\log 7.31}{-\log 2}$$

$$\frac{b \log 2}{\log 2} \leq \frac{\log 7.31 - \log 2}{\log 2}$$

$$b \leq \frac{\log 7.31 - \log 2}{\log 2}$$

$$(15) [4^{2x} = 27] \log$$

$$\cancel{2x} \log 4 = \frac{\log 27}{\cancel{2} \log 4}$$

$$x = \frac{\log 27}{2 \log 4}$$

$$(16) [2^{a-4} = 82.1] \log$$

$$\cancel{(a-4)} \log 2 = \frac{\log 82.1}{\log 2}$$

$$\cancel{a-4} = \frac{\log 82.1}{\log 2}$$

$$a = \frac{\log 82.1}{\log 2} + 4$$

$$(17) [9^{x-2} > 38] \log$$

$$\frac{(x-2) \log 9}{\log 9} > \frac{\log 38}{\log 9}$$

$$\cancel{x-2} > \frac{\log 38}{\log 9}$$

$$x > \frac{\log 38}{\log 9} + 2$$

$$(18) [5^{w+3} = 17] \log$$

$$\frac{(w+3) \log 5}{\log 5} = \frac{\log 17}{\log 5}$$

$$\cancel{w+3} = \frac{\log 17}{\log 5}$$

$$w = \frac{\log 17}{\log 5} - 3$$

$$(19) [30^{x^2} = 50] \log$$

$$\frac{x^2 \log 30}{\log 30} = \frac{\log 50}{\log 30}$$

$$\sqrt{x^2} = \sqrt{\frac{\log 50}{\log 30}}$$

$$x = \sqrt{\frac{\log 50}{\log 30}}$$

$$\textcircled{20} [5^{x^2-3} = 72] \log$$

$$\frac{(x^2-3)\log 5}{\log 5} = \frac{\log 72}{\log 5}$$

$$x^2 - 3 = \frac{\log 72}{\log 5}$$

$$\sqrt{x^2} = \sqrt{\frac{\log 72}{\log 5} + 3}$$

$$x = \sqrt{\frac{\log 72}{\log 5} + 3}$$

$$\textcircled{21} [4^{2x} = 9^{x+1}] \log$$

$$2x(\log 4) = (x+1)\log 9$$

$$2x \log 4 = x \log 9 + \log 9$$

$$-x \log 9 \quad -x \log 9$$

$$2x \log 4 - x \log 9 = \log 9$$

$$x(2 \log 4 - \log 9) = \log 9$$

$$\frac{x(2 \log 4 - \log 9)}{2 \log 4 - \log 9} = \frac{\log 9}{2 \log 4 - \log 9}$$

$$x = \frac{\log 9}{2 \log 4 - \log 9}$$

$$(22) \quad [2^{n+1} = 5^{2n-1}] \log$$

$$(n+1) \log 2 = (2n-1) \log 5$$

$$\begin{array}{r} n \log 2 + \log 2 = 2n \log 5 - \log 5 \\ -2n \log 5 \quad -2n \log 5 \end{array}$$

$$\begin{array}{r} n \log 2 + \log 2 - 2n \log 5 = -\log 5 \\ -\log 2 \quad \quad \quad -\log 2 \end{array}$$

$$n \log 2 - 2n \log 5 = -\log 5 - \log 2$$

$$\frac{n(\log 2 - 2 \log 5)}{\log 2 - 2 \log 5} = \frac{-\log 5 - \log 2}{\log 2 - 2 \log 5}$$

$$n = \frac{-\log 5 - \log 2}{\log 2 - 2 \log 5}$$

$$(23) \quad \frac{\log 12}{\log 5} = 1.5439$$

$$(26) \quad \frac{\log 18}{\log 2} = 4.1699$$

$$(24) \quad \frac{\log 32}{\log 8} = 1.6666$$

$$(27) \quad \frac{\log 6}{\log 9} = 0.8154$$

$$(25) \quad \frac{\log 9}{\log 11} = 0.9163$$

$$(28) \quad \frac{\log \sqrt{8}}{\log 7} = 0.5343$$