AFM Notes
3-4 Common Logarithms

Name____________________
Date______________ Period ______

You have seen that the base 10 logarithm function, $y = \log_{10} x$, is used in many applications. Base 10 logarithms are called Common log. Common logarithms are usually written without the subscript 10.

$\log_{10} x \Rightarrow \log x$

Example 1: Find Common Logarithms

Use a calculator to evaluate each expression to four decimal places.

a. $\log 3 = 0.4771$ $\quad 10^x = 3$

b. $\log 0.2 = -0.6989$ $\quad 10^x = 0.2$

Sometimes an application of logarithms requires that you use the inverse of logarithms, or exponentiation. → re-write as an exponential

Example 2: Solve Logarithmic Equations Using Exponentiation

The amount of energy E, in ergs, that an earthquake release is related to its Richter scale magnitude M by the equation $\log E = 11.8 + 1.5M$. The Chilean earthquake of 1960 measured 8.5 on the Richter scale. How much energy was released?

↳ M

$\log E = 11.8 + 1.5M$

$\log E = 11.8 + 1.5(8.5)$

$\log E = 24.55$

$10^{24.55} = E$

$E = 3.548 \times 10^{24}$ ergs

Example 3: Solve Exponential Equations Using Logarithms

Solve $3^x = 11$

$\log[3^x = 11]\log$

$\frac{x \log 3}{\log 3} = \frac{\log 11}{\log 3}$

$x = 2.1826$

$\log_3 11 = x$

$\frac{\log 11}{\log 3} = x$

B

$\log[5^x = 36]\log$

$\frac{x \log 5}{\log 5} = \frac{\log 36}{\log 5}$

$x = 2.2265$

Example 4: Solve Exponential Inequalities Using Logarithms

Solve $5^{3y} < 8^{y-1}$

$\log[5^{3y} < 8^{y-1}]\log$

$3y \log 5 < (y-1)\log 8$

$3y \log 5 < \cancel{y \log 8} - \log 8$

$-y\log 8 \quad \cancel{-y \log 8}$

$3y\log 5 - y\log 8 < -\log 8$

$\frac{y(3\log 5 - \cancel{\log 8})}{3\log 5 - \cancel{\log 8}} < \frac{-\log 8}{3\log 5 - \log 8}$

$\boxed{y < \frac{-\log 8}{3\log 5 - \log 8}}$

CHANGE OF BASE FORMULA

The change of base allows you to write equivalent logarithmic expressions that have different bases.

Symbols: For all positive numbers, a, b, and n, where $a \neq 1$ and $b \neq 1$,

$$\log_a n = \frac{\log_b n}{\log_b a}$$

Example: $\log_5 12 = \dfrac{\log_{10} 12}{\log_{10} 5}$ $\dfrac{\log 12}{\log 5}$ → changing to base 10 so we can use the calculator

This formula makes it possible to evaluate a logarithmic expression of any base by translating the expression into one that involves common logarithms.

Example 5: Change of Base Formula

Express $\log_4 25$ in terms of common logarithms. Then approximate its value to four decimal places.

$$\log_4 25 = \frac{\log 25}{\log 4} = 2.3219$$