AFM Notes
3-5 Base *e* and Natural Logarithms

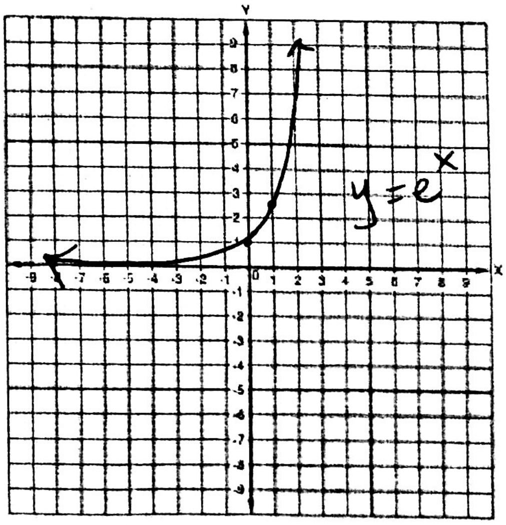
Name________________
Date________________ Period ______

Suppose a bank compounds interest on accounts continuously that is with no waiting time between interest payments. In order to develop an equation to determine continuously compounded interest; examine what happens to the value A of an account for increasingly larger numbers of compounding periods n. Use principal P of \$1, an interest rate r of 100%, and time t of 1 year.

n	$A = P\left(1+\frac{r}{n}\right)^{nt}$	A
1 (yearly)	$A = 1\left(1+\frac{1}{1}\right)^{1(1)}$	2
4 (quarterly)	$A = 1\left(1+\frac{1}{4}\right)^{4(1)}$	2.4414...
12 (monthly)	$A = 1\left(1+\frac{1}{12}\right)^{12(1)}$	2.6130...
365 (daily)	$A = 1\left(1+\frac{1}{365}\right)^{365(1)}$	2.7145...
8760 (hourly)	$A = 1\left(1+\frac{1}{8760}\right)^{8760(1)}$	2.7181...

In the table above, as n increases, the expression $A = 1\left(1+\frac{1}{n}\right)^{n(1)}$ or $A = 1\left(1+\frac{1}{n}\right)^{n}$ approaches the irrational number 2.71828..... This number is referred to as the natural base e.

An exponential function with base e is called a natural base exponential function. The graph of $y = e^x$ is shown at the right. Natural base exponential functions are used extensively in science to model quantities that grow and decay continuously.



X	Y
0	1
1	e(2.71)

Most calculators have an e^x function for evaluating natural base expressions.

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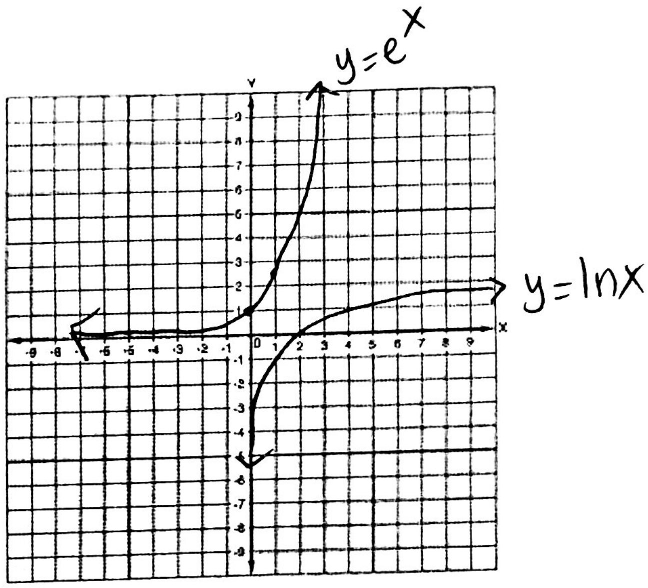
Example 1: Evaluate Natural Base Expressions

Use a calculator to evaluate each expression to four decimal places.

a. e^2 $= 7.3891$

b. $e^{-1.3}$ $= 0.2725$

The logarithm with base e *is* called the natural logarithm sometimes denoted by $\log_e x$, but more often abbreviated $\ln x$. The natural logarithmic function, $y = \ln x$, is the inverse of the natural base exponential function, $y = e^x$. The graph of these two functions shows that $\ln 1 = 0$ and $e = 1$.



Example 2: Evaluate Natural Logarithmic Expressions

LN on the calculator is next to the #4 button

a. $\ln 4$ 1.3864

b. $\ln 0.05$ -2.9957

You can write an equivalent base *exponential* equation for a natural logarithmic equation and vice versa by using the fact that they are just logs.

Example 3: Write Equivalent Expression

$\log_e \Rightarrow \ln$

a. $e^x = 5$

$\log_e 5 = x$

$\ln 5 = x$

b. $\ln x \approx 0.6931$

$\log_e x = 0.6931$

$e^{0.6931} = x$

Since the natural base function and the natural logarithmic function are inverses, these two functions can be used to "undo" each other.

Example 4: Inverse Property of Base e and Natural Logarithms

a. $e^{\ln 7}$ → $e^{\log_e 7} = 7$

b. $\ln e^{4x+3}$ → $\log_e e^{4x+3} = 4x+3$

Equations and inequalities involving base e are easier to solve using natural logarithms than using common logarithms. All of the properties of logarithms that you have learned apply to natural logarithms as well. ← very important!

Example 5: Solve Base e Equations

Solve $5e^{-x} - 7 = 2$.

$$5e^{-x} - 7 = 2$$
$$+7 \quad +7$$

• move any constants away from e

$$\frac{5e^{-x}}{5} = \frac{9}{5}$$

• divide by any coefficients of e

$$\ln\left[e^{-x} = 9/5\right]\ln$$

• natural log both sides

$\ln e \Rightarrow 1$

$$-x \ln e = \ln(9/5)$$
$$-x(1) = \ln(9/5)$$
$$x = -\ln(9/5)$$
$$x = -0.5877$$

When interest is compounded continuously, the amount A in an account after t years is found using the formula $A = Pe^{rt}$, where P is the amount of principal and r is the annual interest rate.

↓ must change percent to a decimal

$25\% = 0.25$

Example 6: Solve Base e Inequalities

Suppose you deposit $1000 in an account paying 5% annual interest, compounded continuously.

$P=1000 \quad r=0.05$

a. What is the balance after 10 years?

$t=10$

$A=1000e^{0.05(10)}$

$\boxed{A=\$1648.72}$

b. How long will it take for the balance in your account to reach at least $1500?

$A=1500 \quad P=1000 \quad r=0.05 \quad t=?$

$\frac{1500}{1000} \le \frac{1000e^{0.05t}}{1000}$

$\ln\left[1.5 \le e^{0.05t}\right]\ln$

$\ln 1.5 \le 0.05t \ln e$

$\frac{\ln 1.5}{0.05} \le \frac{0.05t(1)}{0.05}$

$8.101 \le t$

It would take at least 8 yrs to make $1500

Example 7: Solve Natural Log Equations and Inequalities

a. $\ln 5x = 4$

$\log_e 5x = 4$

$\frac{e^4}{5} = \frac{5x}{5}$

$\boxed{x = 10.9196}$

b. $\ln(x-1) > -2$

$\log_e (x-1) > -2$

$x-1 > e^{-2}$

$x-1 > 0.1353$

$+1 \quad +1$

$x > 1.1353$