**Unit 6 Day 5**

Warm-Up:

1. What is the **midpoint** of the segment with endpoints (3, 4) and (-2, 7)?

$$\left(\frac{3+-2}{2}, \frac{4+7}{2}\right) = \left(\frac{1}{2}, \frac{11}{2}\right)$$

2. What is the **distance** between (8, -7) and (5, 10)?

$$\sqrt{(8-5)^2 + (-7-10)^2}$$

$$\sqrt{9 + 289} = \sqrt{298} \approx \boxed{17.26}$$

3. Simplify: $x(9x - 5y) - x(x-7y) - 6x(3 + 3y)$

$$9x^2 - 5xy - x^2 + 7xy - 18x - 18xy$$

$$\boxed{8x^2 - 16xy - 18x}$$

On December 31st around 10 pm, you are sitting quietly in your house watching Dick Clark's New Year's Eve special when there is a knock at the door. Wondering who could possibly be visiting at this hour you ead to the front door to find out who it is. Seeing a man dressed in a three-piece suit and tie and holding a briefcase, you cautiously open the door. The man introduces himself as a lawyer representing the estate of your recently deceased great uncle. Turns out your uncle left you some money in his will, but you have to make a decision. The man in the suit explains that you have three options for how to receive your allotment.

Option A: $1000 would be deposited on Dec 31st in a bank account bearing your name and each day an additional $1000 would be deposited (until January 31st).

Option B: One penny would be deposited on Dec 31st in a bank account bearing your name. Each day, the amount would be doubled (until January 31st).

Option C: Take $30,000 on the spot and be done with it.

Without "doing the math" first, I would instinctively choose the following option (circle your choice):

A B C

| Option A:<br>$1000 to start + $1000 per day | Option B:<br>$.01 to start then double each day |
|---|---|
| **Note that t = 0 on Dec. 31st** | |

Table of input/output values

| t =<br>time in # of days | A(t)=<br>$ in account after t days |
|---|---|
| 0 | 1000 |
| 1 | 2000 |
| 2 | 3000 |
| 3 | 4000 |
| 4 | 5000 |
| 5 | 6000 |

Table of input/output values

| t =<br>time in # of days | B(t) =<br>$ in account after t days |
|---|---|
| 0 | .01 |
| 1 | .02 |
| 2 | .04 |
| 3 | .08 |
| 4 | .16 |
| 5 | .32 |

The tables above show the values for the first five days for Option A and Option B.

Do you regret your decision??

| | |
|---|---|
| 6 | 7000 |
| 7 | 8000 |
| 8 | 9000 |
| 9 | 10,000 |
| 10 | 11,000 |
| 11 | 12,000 |
| 12 | 13,000 |
| 13 | 14,000 |
| 14 | 15,000 |
| 15 | 16,000 |
| 16 | 17,000 |
| 17 | 18,000 |
| 18 | 19,000 |
| 19 | 20,000 |
| 20 | 21,000 |
| 21 | 22,000 |
| 22 | 23,000 |
| 23 | 24,000 |
| 24 | 25,000 |
| 25 | 26,000 |
| 26 | 27,000 |
| 27 | 28,000 |
| 28 | 29,000 |
| 29 | 30,000 |
| 30 | 31,000 |
| 31 | 32,000 |

| | |
|---|---|
| 6 | .64 |
| 7 | 1.28 |
| 8 | 2.56 |
| 9 | 5.12 |
| 10 | 10.24 |
| 11 | 20.48 |
| 12 | 40.96 |
| 13 | 81.92 |
| 14 | 163.84 |
| 15 | 327.68 |
| 16 | 655.36 |
| 17 | 1,310.72 |
| 18 | 2,621.44 |
| 19 | 5,242.88 |
| 20 | 10,485.76 |
| 21 | 20,971.52 |
| 22 | 41,943.04 |
| 23 | 83,886.08 |
| 24 | 167,772.16 |
| 25 | 335,544.32 |
| 26 | 671,088.64 |
| 27 | 1,342,177.28 |
| 28 | 2,684,354.56 |
| 29 | 5,368,709.12 |
| 30 | 10,737,418.24 |
| 31 | 21,474,836.48 |

WOWWWWW!!!!!!!

# Introduction to Exponential Functions

*New Year's Eve Surprise!*

Why does Option B grow so much faster than Option A? multiplication

As time increases from day 20 to 21, describe how the outputs change for each function

Option A: add 1,000 Option B: almost doubles!

As time increases from day 23 to 24, describe how the outputs change for each function

Option A: add 1,000 Option B: almost doubles!

In general, as each day passes, the outputs for each function...

Option A: add 1,000 Option B: double

Option A is an example of a linear function because you are *adding*.

Option B is an example of an exponential function because you are *multiplying*.

Does the table of values represent a linear function or an exponential function? Explain.

| x | -2 | -1 | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|
| y | $\frac{1}{9}$ | $\frac{1}{3}$ | 1 | 3 | 9 | 27 | 81 |

multiply by 3 ∴ exponential

| x | -2 | -1 | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|
| y | 2 | 2.5 | 3 | 3.5 | 4 | 4.5 | 5 |

add 0.5 ∴ linear

Exponential Functions are of the form $f(x) = a(b^x)$ $f(x) = ab^x$

a = INITIAL VALUE

b = BASE (also called the growth or decay factor)

x represents the INPUT

f(x) or y represents the OUTPUT

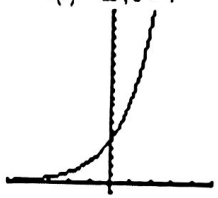
the y-intercept occurs at (0, a) and the base b represents the common ratio as seen with geometric sequences

if the base is greater than 1, the function is exponential growth

if the base is between 0 and 1, the function is exponential decay

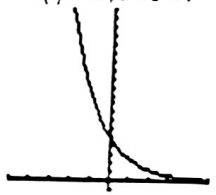
**Graph of a generic Exponential Growth Function**

$f(x) = ab^x, b > 1$



**Graph of a generic Exponential Decay Function**

$f(x) = ab^x, 0 < b < 1$



What do you know?

a) $f(x) = 2(3)^x$ y-int: (0,2) r=3 growth

b) $g(x) = 1523(1.05)^x$

y-int: (0,1523) r=1.05 growth

c) $h(x) = 256(0.85)^x$

y-int: (0,256) r=0.85 decay

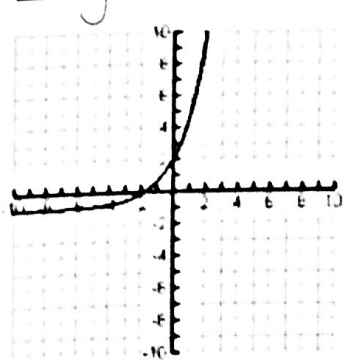
d) $k(x) = 32(0.956)^x$

y-int: (0,32) r=0.956 decay

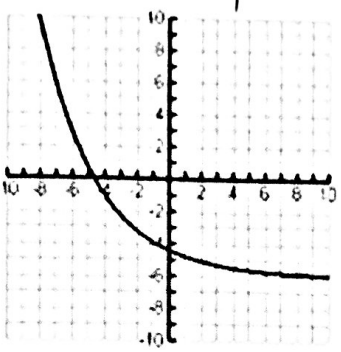
# Exponential Functions

***Determine whether each function is exponential growth or exponential decay.***

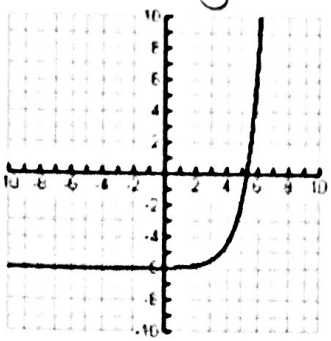
1. growth



2. decay



3. growth



4. decay

$y = 3(0.75)^x$

5. growth

$y = 2.4(1.07)^x$

6. growth

$y = \frac{5}{8}(3)^x$

7. decay

$y = 8\left(\frac{6}{7}\right)^x$

8. decay

$y = \frac{4}{3}\left(\frac{1}{9}\right)^x$

9. growth

$y = 3.14(2.84)^{5x}$

10. Suppose an investment of $10,000 doubles in value every 13 years. How much is the investment worth after 52 years?

$10{,}000(2)^4 = 160000$

11. A population of 100 insects triples in size every month. Write an equation and find how many insects there are after 5 months.

$100(3)^5 = 24{,}300$

12. A tennis match begins with 64 players. Each round, half of the players are eliminated. Write an equation and find out when there will be a winner (i.e., one player left).

$64(1/2)^x = 1$ 64, 32, 16, 8, 4, 2, 1 6 rounds

13. You drop a ball from a height of 12 feet. Each path has 3/5 the height of the previous path. Write equation and find the height at the top of the fourth path.

$12(3/5)^4 = 1.552$ ft

# Day 5 Homework Worksheet

1. Complete the table for the function $f(x) = 3(5)^x$

| X | Y |
|---|---|
| -3 | 0.024 |
| -2 | 0.12 |
| 0 | 3 |
| 1 | 15 |
| 3 | 375 |

2. Simplify the following

a) $(w^2v)^3$ $w^6v^3$

b) $(5c^{-4})(-4m^2c^8)$ $-20m^2c^4$

c) $\frac{21x^3y}{3xy^4}$ $\frac{7x^2}{y^3}$

d) $(12x^2y^{-2})^2$ $144x^4y^{-4} = \frac{144x^4}{y^4}$

e) $\left(\frac{5a^8}{10a^6}\right)^{-3}$ $\left(\frac{a^2}{2}\right)^{-3} = \left(\frac{2}{a^2}\right)^3 = \boxed{\frac{8}{a^6}}$

3. Identify each sequence as arithmetic or geometric. Write the NEXT-NOW form and explicit form for each.

a) 1600, 400, 100, 25, ...

divide by 4 mult. by 1/4

geometric

next = now / 4 next = 1/4 now

b) 14, 21, 28, 35, ...

add 7

arithmetic

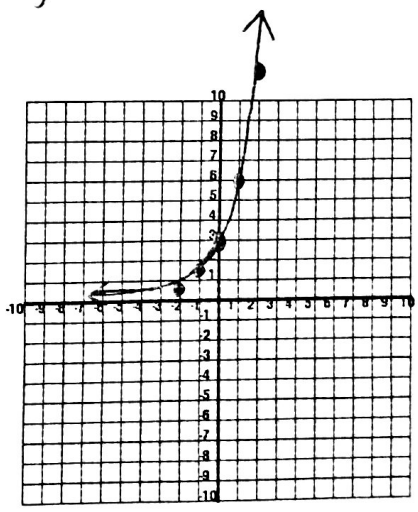
next = now + 7

4. Write an example of an exponential growth equation and an exponential decay equation. How do you know which is which?

growth $y = 4(10)^x$ decay $y = 4(1/10)^x$

5. The function $y = 25(0.80)^x$ models the amount of a 25 mg dose of medicine remaining in the bloodstream after x hours. How many mg of medicine remain after 5 hours?

$25(.80)^5 = \boxed{8.192 \text{ mg}}$

Sketch a graph of $y = 3(2)^x$

| X | Y |
|---|---|
| -2 | 0.75 |
| -1 | 1.5 |
| 0 | 3 |
| 1 | 6 |
| 2 | 12 |